Cambridge O Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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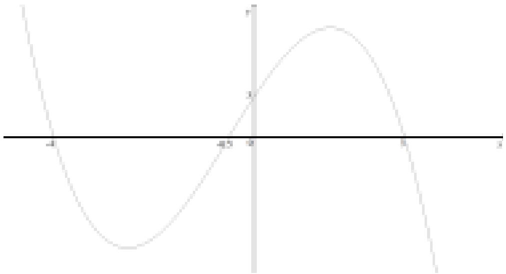
Question	Answer	Marks	Guidance
1		3	**B1** for a cubic shape with a maximum in the first quadrant, a minimum in the third quadrant, extending into the second and 4th quadrants. The extensions must not curve incorrectly and not lead to a complete stationary point. **B1** for x-intercepts $-4, -\frac{1}{2}, 3$ either on diagram or stated but must be with a cubic graph. **B1** for y-intercept 3 either on diagram or stated but must be with a cubic graph.
2	$v = -4.91$ soi	**B1**	
	Speed = 4.91	**B1**	
3	$\tan\left(2x - \frac{\pi}{3}\right) = \pm\sqrt{3}$ soi or $\sin\left(2x - \frac{\pi}{3}\right) = \pm\frac{\sqrt{3}}{2}$ soi	**B1**	**B0** if negative root is rejected Allow truncated decimals May be implied by subsequent work From use of $\operatorname{cosec}^2\left(2x - \frac{\pi}{3}\right) - 1 = \cot^2\left(2x - \frac{\pi}{3}\right)$
	$2x - \frac{\pi}{3} = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ $2x = 0, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}$ $x = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}$ or 0, 1.05, 1.57, 2.62 or greater accuracy	4	**M1** for correct order of operations to obtain one solution in the range using $\tan\left(2x - \frac{\pi}{3}\right) = k$ or $\sin\left(2x - \frac{\pi}{3}\right) = m,\ \lvert m\rvert < 1$ **Dep M1** for correct order of operations to obtain a second solution in the range using $\left(2x - \frac{\pi}{3}\right) = \tan^{-1}(k) \pm \pi$ or $\left(2x - \frac{\pi}{3}\right) = \pi - \sin^{-1}(m),\ \lvert m\rvert < 1$ oe $\left(2x - \frac{\pi}{3}\right) = -\sin^{-1}(m),\ \lvert m\rvert < 1$ oe **A1** for any pair of correct solutions **A1** for remaining pair of solutions, with no extra solutions within the range
4(a)	$\frac{1}{256} - \frac{x^2}{24} + \frac{7x^4}{36}$	3	**B1** for $\frac{1}{256}$ **B1** for $-\frac{x^2}{24}$ **B1** for $\frac{7x^4}{36}$

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Question	Answer	Marks	Guidance
4(b)	$4x^2+4+\frac{1}{x^2}$ soi	B1	
	Coefficient of x^2 $\left(their\,4\times their\,\frac{1}{256}\right)$ $+\left(their\,4\times their\,-\frac{1}{24}\right)$ $+\left(their\,\frac{7}{36}\right)$	M1	Allow one sign error, but must have 3 terms in x^2 only, with an attempt at addition.
	$\frac{25}{576}$	A1	
5(a)	$\frac{a\left(r^4-1\right)}{r-1}=17\frac{a\left(r^2-1\right)}{r-1}$	M1	Allow equivalents Allow if 'a' terms missing (assume to have been cancelled)
	$\left(r^2-1\right)\left(r^2+1\right)=17\left(r^2-1\right)$ or better $r^4-17r^2+16=0$ oe $r^3+r^2-16r-16=0$ oe	M1	**Dep M1** for a correct simplified equation in r only
	$r=4$ only, from correct working	A1	
5(b)	$ar^5=64$	M1	For use of ar^5 with *their* positive r
	$a=0.0625$ or $\frac{1}{16}$	A1	Must be exact A0 if $r=4$ not from correct working in **(a)**
5(c)	Because $r>1$ oe	B1	**FT** on *their* $r>1$ Must have a value for r

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Question	Answer	Marks	Guidance
6(a)	**Either** Starts with 8: 1680	**B1**	1680 must not be part of a product. May be implied by final answer
	Starts with 7 or 9: 2688	**B1**	May be implied by final answer
	Total: 4368	**B1**	
	Or Alternative 1 Starts with 7, 8 or 9 and ends in 1, 3 or 5: 3024	**(B1)**	Allow for 1008 three times May be implied by final answer
	Starts with 8 or 9 and ends in 7: 672 Starts with 7 or 8 and ends in 9: 672	**(B1)**	For both May be implied by final answer
	Total: 4368	**(B1)**	
	Or Alternative 2 13 ways of obtaining odd 5-digit numbers which start with 7, 8 or 9	**(B1)**	Needs to be part of a product. May be implied by final answer
	${}^{8}\mathrm{P}_3$ ways of arranging the remaining 3 digits: 336	**(B1)**	Needs to be part of a product. May be implied by final answer
	Total = $13\times336=4368$	**(B1)**	
	Or Alternative 3 Last digit is 7 or 9: 1344	**B1**	May be implied by final answer
	Last digit is 1, 3 or 5: 3024	**B1**	May be implied by final answer
	Total: 4368	**B1**	
	Or Alternative 4 ${}^{10}\mathrm{P}_5-\left({}^{9}\mathrm{P}_4\times7\right)-\left({}^{8}\mathrm{P}_3\times5\right)-\left({}^{8}\mathrm{P}_3\times4\right)$ $-\left({}^{8}\mathrm{P}_3\times5\right)$	**B2**	Must be complete
	Total: 4368	**B1**	
6(b)	$\frac{n!}{(n-3)!3!}=\frac{2n!}{(n-2)!2!}$ soi	**B1**	
	$(n-2)=6$ soi	**B2**	**Dep B1** on first **B** for $(n-2)$ soi **Dep B1** on first **B** for 6 soi
	$n=8$	**B1**	**Dep** on previous **B** marks

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Question	Answer	Marks	Guidance
7(a)	$\sin AOQ = \frac{7}{10}$ $POA = \pi - AOQ$ or $14^2 = 10^2 + 10^2 - 200\cos AOB$ oe $POA = \frac{2\pi - AOB}{2}$	**M1**	Allow alternatives, but must be a complete method to find *POA*
	$POA = 2.366195157 = 2.366$ to 3 dp	**A1**	Must see an angle correct to more than 3dp used in order to justify 3 dp
7(b)	Area of sector = $\frac{1}{2}10^2(2.366)$ (118.3)	**B1**	Allow unsimplified. Also allow use of 2.37
	Area of triangle = $\frac{1}{2}10^2 \sin 2.366$ (35)	**B1**	Allow unsimplified. Also allow use of 2.37
	Total area = awrt 153	**B1**	Allow greater accuracy
7(c)	Major arc *PB* = 10×2.366	**B1**	Allow unsimplified. Also allow use of 2.37
	$\sin\frac{POA}{2} = \frac{AP/2}{10}$ or $AP^2 = 10^2 + 10^2 - 200\cos POA$	**M1**	For a valid attempt to find *AP* – may be seen in **(a)** or **(b)** but *AP* must be stated in this part.
	$AP = 18.5$	**A1**	Allow awrt 18.5
	Perimeter: major arc *PB* + 20 + *their AP*	**B1**	For plan, may be implied, but must have an attempt to calculate *AP*
	Total perimeter = 62.2	**A1**	Allow awrt 62.2
8(a)	$2x^2 + 2x - 2 = x^2 + 6x - 2$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0,\ x = 4$	**M1**	For obtaining an equation in one variable
		M1	**Dep** for a correct attempt to obtain at least one solution
	$(0, -1)$	**A1**	nfww
	$(4, 19)$	**A1**	nfww
	Mid-point $(2, 9)$ with sufficient detail	**B1**	**AG**

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Question	Answer	Marks	Guidance
8(b)	**Either** Gradient of perpendicular $= -\frac{1}{5}$	**M1**	
	$y - 9 = -\frac{1}{5}(x - 2)$	**M1**	**Dep** on previous **M** mark for perpendicular bisector using *their* mid-point and *their* perpendicular gradient
	$7 - 9 = -\frac{1}{5}(12 - 2)$ oe	**A1**	For checking by substitution, must see evidence.
	Or Alternative 1 Gradient of perpendicular $= -\frac{1}{5}$	**(M1)**	
	$y - 7 = -\frac{1}{5}(x - 12)$	**(M1)**	**Dep** on previous **M** mark for perpendicular bisector using $(12, 7)$ and *their* perpendicular gradient
	$9 - 7 = -\frac{1}{5}(2 - 12)$ oe	**(A1)**	For checking by substitution, must see evidence
	Or Alternative 2 Gradient of perpendicular $= -\frac{1}{5}$	**(M1)**	
	Gradient of line joining *their* $(2, 9)$ to $(12, 7)$ $= = -\frac{1}{5}$	**(M1)**	
	$(2, 9)$ is a common point and gradients of perpendicular bisector and *l* are the same so *C* lies on *l*.	**(A1)**	
8(c)	$(22, 5)$	**2**	**B1** for 22 **B1** for 5
	$(-18, 13)$	**2**	**B1** for -18 **B1** for 13

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Question	Answer	Marks	Guidance
9(a)	$e^{2y} = mx^2 + c$	**B1**	May be implied by later work
	Either $7.96 = 4m + c$ $3.76 = 2m + c$	**M1**	
	$m = 2.1$ oe	**A1**	
	$c = -0.44$ oe	**A1**	
	$y = \frac{1}{2}\ln\left(2.1x^2 - 0.44\right)$ oe	**A1**	Do not isw
	Or gradient = 2.1 oe	**(B1)**	
	Use of either $7.96 = 4m + c$ or $3.76 = 2m + c$	**(M1)**	For use with *their m*
	$c = -0.44$ oe	**(A1)**	
	$y = \frac{1}{2}\ln\left(2.1x^2 - 0.44\right)$ oe	**(A1)**	Must be bracketed correctly
9(b)	$y = \frac{1}{2}\ln\left(\textit{their}\,2.1x^2 - \textit{their}\,0.44\right)$ oe	**M1**	Must use the form $y = k\ln\left(px^2 \pm q\right)$ $p \neq 1$ and $q \neq 0$ or $e^{2y} = mx^2 + c$
	0.253	**A1**	
9(c)	$\textit{their}\,2.1x^2 - \textit{their}\,0.44 > 0$ or $= 0$ or ≥ 0 soi	**B1**	
	Correct attempt to obtain the critical value using $\textit{their}\,2.1x^2 - 0.44 = 0$	**M1**	Must be from the form $y = k\ln\left(px^2 - q\right)$, $p \neq 1$ and $q > 0$
	$x > 0.458$ or $x > \sqrt{\frac{22}{105}}$ oe	**A1**	

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Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx}=(2x+3)^{\frac{1}{2}}+5x(+c)$	**B1**	For $(2x+3)^{\frac{1}{2}}$, allow unsimplified
		M1	For $k(2x+3)^{\frac{1}{2}}+5x$
	$10=3+15+c$	**M1**	**Dep** for use of 10 and $x=3$ in *their* $\frac{dy}{dx}$ to obtain c
	$\frac{dy}{dx}=(2x+3)^{\frac{1}{2}}+5x-8$ soi	**A1**	
	When $x=11$, $\frac{dy}{dx}=5+55-8$ oe $= 52$	**A1**	**AG** – need to see sufficient detail
10(b)	$f(x)=\frac{1}{3}(2x+3)^{\frac{3}{2}}+\frac{5x^2}{2}(-8x+d)$	**B1**	For $\frac{1}{3}(2x+3)^{\frac{3}{2}}$, must be $\int(2x+3)^{\frac{1}{2}}\,dx$
		M1	For $k(2x+3)^{\frac{3}{2}}+\frac{5x^2}{2}$
	$\frac{19}{2}=\frac{27}{3}+\frac{45}{2}-24+d$ $d=2$	**M1**	For use of $y=\frac{19}{2}$ and $x=3$ in *their y*
	$(f(x)=)\ \frac{1}{3}(2x+3)^{\frac{3}{2}}+\frac{5x^2}{2}-8x+2$	**A1**	Allow -8 if obtained from using $\frac{dy}{dx}=52$ in **(a)** rather than $\frac{dy}{dx}=10$
11(a)	$\frac{dy}{dx}=$ $\frac{(x+1)\left(\frac{1}{3}\times 2x\times(x^2-5)^{-\frac{2}{3}}\right)-(x^2-5)^{\frac{1}{3}}}{(x+1)^2}$ or $(x+1)^{-1}\left(\frac{1}{3}\times 2x\times(x^2-5)^{-\frac{2}{3}}\right)$ $+(x^2-5)^{\frac{1}{3}}\left(-(x+1)^{-2}\right)$	**3**	**B1** for $\frac{1}{3}\times 2x\times(x^2-5)^{-\frac{2}{3}}$ **M1** for an attempt at a quotient or a correct product **A1** for all other terms correct
	$\frac{-x^2+2x+15}{3(x+1)^2(x^2-5)^{\frac{2}{3}}}$	**3**	**Dep on first 3 marks** **A1** for $-x^2$ in a quadratic numerator **A1** for $2x$ in a quadratic numerator **A1** for 15 in a quadratic numerator

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Question	Answer	Marks	Guidance
11(b)	$-x^2+2x+15=0$	**M1**	For attempt to solve *their* $-x^2+2x+15=0$ to obtain $x=$ .. Must be a quadratic equation.
	$x=5$ only	**A1**	
11(c)	**Either** Find the gradient either side of the stationary point	**B1**	
	If gradient changes from +ve to -ve: max If gradient changes from -ve to +ve: min	**B1**	**Dep** on previous **B1**
	Or Alternative 1 Take the second derivative and substitute in the value of x obtained in **(b)**	**(B1)**	Allow alternative valid methods Allow a general method as this is not dependent on **(a)** or **(b)**
	If second derivative is + ve, then a min If second derivative is – ve, then a max	**(B1)**	**Dep** on previous **B1**
	Or Alternative 2 Consider a y-value to one side of the stationary point	**(B1)**	Allow alternative valid methods Allow a general method as this is not dependent on **(a)** or **(b)**
	If y value of stationary point is greater, then a max. If y value of stationary point is less, then a min.	**(B1)**	**Dep** on previous **B1**

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